

FIT-HE-96-81
hep-th/xxxxxxx
August 1996

String Loop Corrections and the Condensation of Tachyon in 2d Gravity

K. GHOROKU *

*Fukuoka Institute of Technology
Wajiro, Higashi-ku, Fukuoka 811-02, Japan*

ABSTRACT

Quantum theory of 2d gravity is examined by including a special quantum correction, which corresponds to the open string loop corrections and provides a new conformal anomaly for the corresponding σ model. This anomaly leads to the condensation of the tachyon, and the resultant effective theory implies a possibility of extending the 2d gravity to the case of $c > 1$.

* e-mail address: gouroku@dontaku.fit.ac.jp

1. Introduction

Two dimensional gravity is closely related to the non-critical string theory in the sense that the string world sheet action can be identified with the quantized 2d gravitational theory. In both theories, the vacuum state is determined by the principle of the conformal invariance. In spite of the large development of 2d gravitational theory, there remains a problem how to add a number of matter fields in a way not to destroy the stability of the surface. The study of the vacuum state of the 2d surface so far has been performed through the field equations obtained from the short distance behavior of the fields living on the surface, and we can find the complex dressed factor of the tachyon-perturbation for $c > 1$ [1], where c denotes the central charge. This complex dressed factor implies the instability of the vacuum 2d gravity since the tachyon is corresponding to the cosmological term in the 2d action. This difficulty is known as the $c = 1$ wall problem, and its origin is reduced to the appearance of the true tachyon mode, which leads to the unstable surface fluctuation. Although the properties like the renormalization group equations of matter fields are not sensitive to the vacuum state of the surface [2], we should reconsider the string field equations obtained by a simple analysis of α' expansion [3] in order to resolve the problem of the vacuum of 2d surface.

In order to approach this problem, we consider two effects other than the usual α' -expansions. One is the nonlinear term of the tachyon equation. This is obtained by expanding the tachyon field in its power series and picking up the anomaly [4, 5]. This method is nonperturbative from the viewpoint of α' -expansion, but it would be difficult to resolve $c = 1$ problem by this term only [6, 7]. Another is the effect of the fluctuations of the surface which could be formulated by the string loops [8]. The most simple one is obtained by the one-loop correction of open string whose end point is connected to the boundary on the surface [10]. This state of the surface can be constructed as a boundary state with a tube of the closed string which propagates starting from the surface into the vacuum. This formulation has been given for the case of the superstring or for the critical string, and its technique [10, 9] has recently attracted many attentions since it is applied to the derivation of the D-brane action [11]. Our purpose is here to extend this formulation to the case of the noncritical string to resolve the problem of 2d gravity mentioned above.

The analysis is performed near the linear dilaton vacuum, and we assume that the tachyon field is small and the string coupling constant is also small for the sake of the consistency of the approximation. The guiding principle of our analysis is the conformal invariance or the BRST invariance of the theory. In the estimation of the anomaly for the boundary state with a tube in the noncritical string theory, we should notice the following two points. (i) This tube amplitude badly diverges for $c > 1$, where the tachyonic state appears under the linear dilaton vacuum. (ii) The second point is the appearance of the Liouville field and the background

charge term in the world sheet action.

In order to evade the first point, we consider for $c \leq 1$. And it might be possible to approach the theory of $c > 1$ from the analysis of $c = 1$ case. In fact, we might provide a clue to the extension to this direction. To resolve the second point in our analysis, we propose a small device of the technique so that it is applicable to the non-critical string theory and to the 2d gravity.

2. Tree Vacuum State

We set up a vacuum state to calculate the string loop corrections. Such a vacuum for the string theory is obtained from the principle of the conformal invariance of its world-sheet action. Equivalently, the effective action of the quantized 2d gravity is also given by imposing the same principle on the following non-linear σ -model form of the action,

$$S_{2d} = \frac{1}{4\pi} \int d^2z \sqrt{\hat{g}} \left[\frac{1}{2} G_{\mu\nu}(X) \hat{g}^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu + \hat{R}\Phi(X) + T(X) \right] + \hat{S}_{\text{gh}}, \quad (2.1)$$

$$\hat{S}_{\text{gh}} = \frac{1}{2\pi} \int d^2z \sqrt{\hat{g}} \hat{g}^{\alpha\beta} c^\gamma \nabla_\alpha b_{\beta\gamma}, \quad (2.2)$$

where the conformal gauge is taken, $g_{\alpha\beta} = e^{2\phi} \hat{g}_{\alpha\beta}$ and $\hat{g}_{\alpha\beta}$ is some fiducial metric. The conformal mode (ϕ) and c -scalar fields (x^i) in the world sheet action are denoted by $X^\mu = \{\phi, x^i\}$, where $\mu = 0, i$ and $i = 1 \sim c$. \hat{S}_{gh} represents the ghost-action. The result of α' -expansion and T -expansion so far is summarized by the following target space action,

$$S_T = \frac{1}{4\pi} \int d^d X \sqrt{G} e^{-2\Phi} \left\{ R - 4(\nabla\Phi)^2 + (\nabla T)^2 + v(T) - \tilde{k} \right\}, \quad (2.3)$$

where $\tilde{k} = (25 - c)/3$, $d = 1 + c$ and

$$v(T) = -2T^2 + \frac{1}{6}T^3 + \dots. \quad (2.4)$$

The second term of (2.4) has been obtained by expanding the tachyon term, $\int d^2z \sqrt{\hat{g}} T(X)$, of (2.1) in its series [4, 5]. In each Feynman graph of this expansion, infinite series of α' are included since the fluctuation field is exponentiated in the vertex. Then this expansion is regarded as a non-perturbative procedure from the viewpoint of the α' -expansion. Here, we consider this term simultaneously with the perturbative terms obtained by the α' -expansion since it is non-derivative one. Other higher order terms of T (the dotted part) are suppressed here by assuming $T \ll 1$.

From (2.3), the following linear dilaton vacuum is obtained as a well defined vacuum of the theory at tree level,

$$G_{\mu\nu} = G_{\mu\nu}^{(0)} \equiv \delta_{\mu\nu}, \quad \Phi = \Phi^{(0)} \equiv \frac{1}{2}Q\phi, \quad T = 0, \quad (2.5)$$

where $Q = \sqrt{\tilde{k}}$. Then, S_{2d} can be written as

$$S_{2d} = \frac{1}{8\pi} \int d^2z \sqrt{\hat{g}} \left[\hat{g}^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu + Q \hat{R} \phi \right] + \hat{S}_{gh}, \quad (2.6)$$

Around the vacuum (2.5), we make the mode expansion of the fields X^μ . Differently from the critical string case, we should be careful about the existence of the Liouville mode $\phi = X^0$ which couples to the background charge. For the mode expansion of this field, the Coulomb gas picture is applied [14] and we obtain,

$$X^\mu(z, \bar{z}) = \varphi^\mu(z) + \bar{\varphi}^\mu(\bar{z}), \quad (2.7)$$

$$\partial \varphi^\mu(z) = -i \sum_m \alpha_m^\mu z^{-m-1}, \quad (2.8)$$

$$\bar{\partial} \bar{\varphi}^\mu(\bar{z}) = -i \sum_m \bar{\alpha}_m^\mu \bar{z}^{-m-1}, \quad (2.9)$$

where $z = \exp(\tau + i\sigma)$ and

$$[\alpha_m^\mu, \alpha_n^\nu] = \delta^{\mu\nu} m \delta_{m+n,0}. \quad (2.10)$$

The vacuum for these bosonic fields is defined as,

$$\alpha_m^\mu |0\rangle = \bar{\alpha}_m^\mu |0\rangle = \begin{cases} 0 & \text{for } m > 0 \text{ and } m = 0, \mu = i > 0 \\ -\frac{i}{2}Q & \text{for } m = \mu = 0 \end{cases} \quad (2.11)$$

The ghost fields are expanded as

$$\begin{aligned} c(z) &= \sum_m c_m z^{-m+1}, \\ b(z) &= \sum_m b_m z^{-m-2}, \end{aligned} \quad \{c_n, b_m\} = \delta_{n+m,0}. \quad (2.12)$$

Similar formula are obtained for $\bar{b}(\bar{z})$ and $\bar{c}(\bar{z})$. The vacuum of the ghost would be given in the next section. And the stress tensors for each field are obtained as follows,

$$T^\phi(z) = -\frac{1}{2} : \partial \phi \partial \phi : + \frac{Q}{2} \partial^2 \phi, \quad (2.13)$$

$$T^X(z) = \sum_{i=1}^c -\frac{1}{2} : \partial X^i \partial X^i :, \quad (2.14)$$

$$T^{bc}(z) = : c \partial b + 2 \partial c b :. \quad (2.15)$$

These formula are used to construct the boundary state in the next section.

3. Boundary State

The higher genus manifold is constructed by connecting the one of lower genus in terms of tubes of the closed string. Then the tube with the boundary on the surface can be regarded as the elementary stuff of a complicated 2d manifold. We construct this tube state in terms of the loop of the open string whose end point is sewing the boundary of the world surface. Another end point disappears in the vacuum or couples to the external string configurations. The boundary state with a tube made in this way is not conformal invariant and it leads to a conformal anomaly. We consider this formulation for the noncritical string case given above.

First, consider the boundary, which is set at time τ , where the following conditions for the end point of the open string are demanded,

$$\left. \frac{\partial X_i}{\partial \tau} \right|_{\tau} = 0, \quad \text{for } i = 1 \sim c, \quad (3.1)$$

$$\left. \frac{\partial \phi}{\partial \tau} \right|_{\tau} = -\frac{i}{2}Q, \quad (3.2)$$

where $\phi = X^0$ and

$$Q = \sqrt{\frac{25-c}{3}}, \quad (3.3)$$

as given above. The second condition (3.2) for the Liouville field is taken such that it is consistent with (2.11). And this implies a definite form of the noncritical open string action since these boundary conditions should be derived from the effective action of the noncritical open string. The Liouville action for the open string has firstly given by [12], and a more desirable action being consistent with the idea of [1] can be obtained as follows [13],

$$S_{\phi}^{\text{eff}} = S_L + \frac{1}{4\pi} \int_{\partial M} d\hat{s} K_{\hat{g}} Q \phi, \quad (3.4)$$

where $K_{\hat{g}}$ denotes the extrinsic curvature on the boundary ∂M and

$$S_L = \frac{1}{4\pi} \int d^2z \sqrt{\hat{g}} \left[\frac{1}{2} \hat{g}^{\alpha\beta} \partial_{\alpha} \phi \partial_{\beta} \phi + \frac{1}{2} Q \hat{R} \phi \right]. \quad (3.5)$$

We notice here that S_L is common to the closed string case and the background charge on the boundary is twice the one on the surface. The second point is needed by the consistency with the Gauss-Bonnet theorem. The condition (3.2) is obtained if we choose the fiducial metric such as $\hat{g}_{\alpha\beta} = \delta_{\alpha\beta} e^{\varphi}$ and $\partial_n \varphi = -i/2$. The scalar (X^i) part has no boundary term. Then the total effective action for the open string is written as,

$$S^{\text{eff}} = \frac{1}{4\pi} \int d^2z \sqrt{\hat{g}} \left[\frac{1}{2} \hat{g}^{\alpha\beta} \partial_{\alpha} X^i \partial_{\beta} X^i \right] + S_{\phi}^{\text{eff}}. \quad (3.6)$$

Then the conditions (3.1) and (3.2) lead to the following operator relations,

$$\begin{aligned}\alpha_{-m}^\mu &= -\bar{\alpha}_m^\mu e^{-2m\tau} \quad \text{for } m \neq 0, \\ \alpha_0^i &= \bar{\alpha}_0^i = 0, \quad \alpha_0^0 = \bar{\alpha}_0^0 = -\frac{i}{2}Q.\end{aligned}\tag{3.7}$$

The boundary state which is consistent with the above equations is obtained as,

$$|B\rangle_{\text{boson}} = \exp\left(\sum_{m=1}^{\infty} e^{2m\tau} \alpha_m^\mu \bar{\alpha}_m^\mu\right) |0\rangle, \tag{3.8}$$

where the bosonic vacuum is defined in (2.11).

The constraints for the ghost operators, c_m and b_m , are derived from the requirement of the BRST invariance of the boundary state $|B\rangle$ [10]. The BRST operator is defined as

$$\begin{aligned}d &= \frac{1}{2\pi i} \oint J(z) dz, \\ J(z) &= : [T^\phi(z) + T^X(z) + \frac{1}{2}T^{bc}(z)] c(z) :, \end{aligned}\tag{3.9}$$

where the stress tensors are given in (2.13) \sim (2.15). And \bar{d} can be obtained similarly. The BRST invariance for the boundary state is represented as

$$(d + \bar{d})|B\rangle = 0, \tag{3.10}$$

and this is satisfied for

$$d = -\bar{d}. \tag{3.11}$$

Denoting the explicit form of the operator d as

$$d = \sum_n (L_n^\phi + L_n^X) c_{-n} - \frac{1}{2} \sum_{n,m} (n-m) : c_{-n} c_{-m} b_{n+m} : - c_0, \tag{3.12}$$

$$L_n^\phi = \frac{1}{2} \sum_m : a_m^0 a_{n-m}^0 : + \frac{i}{2} Q(n+1) a_n^0, \tag{3.13}$$

$$L_n^X = \frac{1}{2} \sum_{i,m} : a_m^i a_{n-m}^i :, \tag{3.14}$$

and noticing the relation $L_{-n} = \bar{L}_n$, we find that (3.11) is realized by

$$c_n = -\bar{c}_{-n}, \quad b_n = \bar{b}_{-n}. \tag{3.15}$$

Then the boundary state with respect to the ghost can be obtained as

$$|B\rangle_{gh} = \exp\left\{\sum_{n=1}^{\infty} e^{2n\tau} [\bar{c}_{-n} b_{-n} + c_{-n} \bar{b}_{-n}]\right\} (c_0 + \bar{c}_0) |\downarrow\downarrow\rangle, \tag{3.16}$$

where $|\downarrow\downarrow\rangle$ is the Siegel vacuum which satisfies

$$\begin{aligned} b_n|\downarrow\downarrow\rangle &= 0 & \text{for } n \geq 0 \\ c_n|\downarrow\downarrow\rangle &= 0 & \text{for } n \geq 1 \\ \langle\uparrow\uparrow|b_n &= 0 & \text{for } n \leq -1 \\ \langle\uparrow\uparrow|c_n &= 0 & \text{for } n \leq 0 \end{aligned} \quad (3.17)$$

and $\langle\uparrow\uparrow|\downarrow\downarrow\rangle = 1$. We can find the relations (3.15) for $n > 0$ from the above right eigenvector. The relations for $n < 0$ are found from the following left eigen vector,

$$\langle\uparrow\uparrow|(b_0 - \bar{b}_0) \exp \left\{ \sum_{n=1}^{\infty} e^{2n\tau} [\bar{c}_n b_n + c_n \bar{b}_n] \right\}, \quad (3.18)$$

However, we find the right and left cylinder vacuums are orthogonal,

$$\langle\uparrow\uparrow|(b_0 - \bar{b}_0)(c_0 + \bar{c}_0)|\downarrow\downarrow\rangle = 0. \quad (3.19)$$

Then we must demand that the propagator should be accompanied with the zero modes of b and/or c in order to get a nonzero cylinder amplitude which is sandwiched by the boundary states. The appropriate zero mode which is attached to the propagator of the closed string is the following combination [10], $-(b_0 + \bar{b}_0)$. Finally, we arrive at the following boundary state with a tube of the closed string,

$$|\Psi\rangle_B = -(b_0 + \bar{b}_0)[L_0 + \bar{L}_0 - 2]^{-1} \{|B\rangle + |C\rangle\}, \quad (3.20)$$

where $|C\rangle$ denotes the boundary state of Mobius strip which gives a different normalization coefficient from that of the annulus $|B\rangle$. This point is not important here, but the details is seen in [10].

It is easy to see that the state (3.20) leads to the anomaly. Using the relation,

$$[d + \bar{d}, b_0 + \bar{b}_0] = L_0 + \bar{L}_0 - 2, \quad (3.21)$$

we obtain

$$(d + \bar{d})|\Psi\rangle_B = |B\rangle_0 + |C\rangle_0, \quad (3.22)$$

where the index 0 in $|B\rangle_0 + |C\rangle_0$ denotes the zero-mass closed string state in $|B\rangle$ and $|C\rangle$. For the case of the super-symmetric critical string, the massless states are gravitons and the dilaton, so the anomaly (3.22) provides the modified equations of motion for these fields. However, we here concentrate on the case of the non-critical string. In this case, the tachyon is the ground state and its mass becomes imaginary for $c > 1$, and this implies that the amplitude diverges badly and the anomaly is not well defined. Then we consider the case of $c = 1$, where tachyon is massless. Then we obtain

$$|B\rangle_0 + |C\rangle_0 = \kappa(c_0 + \bar{c}_0)|\downarrow\downarrow\rangle, \quad (3.23)$$

where κ denotes the product of the string coupling constant and the numerical factor depending on the details of the open string model considered here. But its precise value is not necessary here, so we do not consider on this point. We assume only $\kappa \ll 1$, because we are considering in the small coupling region where perturbation with respect to the string loop expansion is valid.

4. Equation of Motion

The conditions of conformal invariance of (2.1) are summarized by the target space action (2.3) for the case without the string loop corrections. As shown in the previous section, string loop corrections provide new kind of conformal anomaly. These anomalies do not cancel within themselves except for a special case of the superstring model. According to [10], we assume here the two contributions, tree and the loop, cancel the anomaly each other. Then the target space action should be modified by adding the term coming from string loop corrections for getting more accurate equations.

In order to obtain the appropriate effective action, we derive the equations of string fields according to the operator formalism for the non-critical string of $c = 1$. Consider two kinds of fluctuations around the dilaton vacuum. One is the local field fluctuations on the surface, and it is denoted by $|\Psi >_T$,

$$|\Psi >_T = \left\{ T(x) + h_{\mu\nu}(x)\bar{\alpha}_{-1}^\mu\alpha_{-1}^\nu + \tilde{\Phi}(x)[\bar{c}_{-1}b_{-1} + c_{-1}\bar{b}_{-1}] \right\} |\downarrow\downarrow>. \quad (4.1)$$

The other is the boudary state with the tube of the closed string which is obtained in the previous section and is denoted by $|\Psi >_B$ (see (3.20)). Then we consider the superposition of these state,

$$|\Psi > = |\Psi >_T + |\Psi >_B. \quad (4.2)$$

And, the following equations are obtained by imposing the BRST invariance on this state,

$$\begin{aligned} (d + \bar{d})|\Psi > &= \left[\left(\frac{1}{2}p^2 + \frac{1}{8}Q^2 - 1 \right) T + \left(\frac{1}{2}p^2 + \frac{1}{8}Q^2 \right) h_{\mu\nu}\bar{\alpha}_{-1}^\mu\alpha_{-1}^\nu \right. \\ &+ \left. \left(\frac{1}{2}p^2 + \frac{1}{8}Q^2 \right) \tilde{\Phi}(\bar{c}_{-1}b_{-1} + c_{-1}\bar{b}_{-1}) \right] (c_0 + \bar{c}_0) |\downarrow\downarrow> \\ &+ \left[[p^\mu h_{\mu\nu} + \frac{i}{2}Qh_{0\nu} - p_\nu\tilde{\Phi}]\alpha_{-1}^\nu\bar{c}_{-1} + \frac{i}{2}Q\tilde{\Phi}\alpha_{-1}^0\bar{c}_{-1} \right. \\ &+ \left. [p^\nu h_{\mu\nu} + \frac{i}{2}Qh_{\mu 0} - p_\mu\tilde{\Phi}]\bar{\alpha}_{-1}^\nu c_{-1} + \frac{i}{2}Q\tilde{\Phi}\bar{\alpha}_{-1}^0 c_{-1} \right] |\downarrow\downarrow> \\ &+ \kappa(c_0 + \bar{c}_0) |\downarrow\downarrow> = 0, \end{aligned} \quad (4.3)$$

From this formula, we obtain the following equations,

$$(-\bar{\nabla}^2 + \frac{1}{4}Q^2 - 2)T = \kappa, \quad (4.4)$$

$$(-\bar{\nabla}^2 + \frac{1}{4}Q^2)h_{\mu\nu} = (-\bar{\nabla}^2 + \frac{1}{4}Q^2)\tilde{\Phi} = 0. \quad (4.5)$$

where $\bar{\nabla}^2 = \sum_{i=1}^c \partial_i^2 + (\partial_0 - Q/2)^2$ and the gauge fixing conditions,

$$\partial^\mu h_{\mu\nu} = Qh_{0\nu} + \partial_\nu \tilde{\Phi}. \quad (4.6)$$

Here we should notice that the zeroth component of the momentum in (4.3) has the following correspondence, $p^0 = \alpha^0 + iQ/2$. On the other hand, the differential operator has the correspondence, $\partial^0 = i\alpha^0$.

These equations are obtained from the following target space action,

$$S_T^{\text{eff}} = S_T + 2\kappa \frac{1}{4\pi} \int d^d X \sqrt{G} e^{-\Phi} \tilde{T}, \quad (4.7)$$

where S_T is given in (2.3) and

$$\tilde{T} = \exp(\Phi^{(0)})T. \quad (4.8)$$

Here $\Phi^{(0)}$ is given in (2.5). It should be noticed that the added part in S_T^{eff} is dependent on the tree vacuum $(\Phi^{(0)})$. This is reasonable since the boundary state and the string propagator are intimately related to the background configurations.

In fact, the equations (4.4) and (4.5) can be obtained by the following equations, which are derived from S_T^{eff} ,

$$\nabla^2 T - 2\nabla\Phi\nabla T = \frac{1}{2}v'(T) + \kappa e^{\Phi-\Phi^{(0)}}, \quad (4.9)$$

$$\nabla^2 \Phi - 2(\nabla\Phi)^2 = -\frac{\tilde{k}}{2} + \frac{1}{2}v(T) - \frac{2+d}{8}\kappa e^{\Phi}\tilde{T}, \quad (4.10)$$

$$-R_{\mu\nu} = -2\nabla_\mu\nabla_\nu\Phi + \nabla_\mu T\nabla_\nu T + \frac{\kappa}{4}e^{\Phi}\tilde{T}G_{\mu\nu}. \quad (4.11)$$

where $v' = dv/dT$, and ∇_μ denotes the covariant derivative with respect to the metric G_{ab} . We can solve these equations by keeping the terms of order κ and assuming T is small. The result is given by

$$G_{\mu\nu} = \delta_{\mu\nu}, \quad \Phi = \frac{1}{2}Q\phi, \quad T = T^{(1)} \equiv \frac{1}{2}\kappa. \quad (4.12)$$

The value of T is shifted by the order of κ . This solution implies the occurrence of the condensation of the tachyon due to the string loop corrections. On the other hand, $G_{\mu\nu}$ and Φ do not get any $O(\kappa)$ corrections. This is understood from eqs.(4.10),(4.11).

5. Stability of 2d Surface

As is well-known, the vacuum obtained for $\kappa = 0$ has been unstable for $c > 1$. This is because of the appearance of the true tachyon with negative mass-square. In [5], a proposal has been given to resolve this difficulty. If we seriously consider the term T^3 in the potential $v(T)$, then a non-trivial minimum is found at $T(= T_0) = 8$ for

$$v(T) = -2T^2 + T^3/6. \quad (5.1)$$

Then there is no unstable fluctuation at this new vacuum point, and the authors in [5] has used this potential to explain the rolling of the universe from a false vacuum ($T = 0$) to the true one ($T = T_0$).

Although this story of the cosmology might be true, it would be unreasonable to believe the point ($T = T_0$) as the true vacuum. Because the value T_0 is too large to be able to approximate the potential of T by (5.1) near this point. Around this point, we should need more higher order terms of T . Then, it would be dangerous to say some definite statement in terms of (5.1). A similar situation is seen in the case of the $\lambda\phi^4$ field theory, where we find a non-trivial minimum of the one-loop corrected effective potential. However, this minimum could not be related to the true vacuum since this point is far from the perturbation. While, a new situation has been opened when the system is changed to a gauge theory with a complex scalar. In this case, the loop corrections of the gauge fields provides a minimum within the perturbation. Then this minimum implies a true vacuum of the theory, and this is known as the Coleman-Weinberg mechanism [15]. We can see here a similar situation for T -condensation.

Since T is assumed to be small in obtaining $v(T)$, the term T^3 was neglected in solving (4.9) and we obtained the solution (4.12) by including the correction of $O(\kappa)$. This solution does not have nothing to do with the minimum of the potential (5.1). But this does not mean that the term T^3 is meaningless, because this term plays an important role when we study the stability of the vacuum which is corrected by $O(\kappa)$.

A way to see the stability of the surface is to examine the dressed factor of the perturbation for the vacuum as done in the case of $\kappa = 0$, where the perturbation of the tachyon has been studied. Here the following perturbations are considered,

$$G_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}, \quad \Phi = \Phi^{(0)} + \Phi^{(1)}, \quad T = \frac{1}{2}\kappa + \tilde{t}. \quad (5.2)$$

Then, the next equations linearized with respect to $h_{\mu\nu}$, ϕ and \tilde{t} are obtained from eqs.(4.9) \sim (4.11),

$$(-\bar{\nabla}^2 + \frac{1}{4}Q^2 - [2 - \frac{\kappa}{4}])\tilde{t} = \kappa\phi, \quad (5.3)$$

$$(-\bar{\nabla}^2 + \frac{1}{4}Q^2)h_{\mu\nu} = \kappa\delta_{\mu\nu}\tilde{t}, \quad (5.4)$$

$$(-\bar{\nabla}^2 + \frac{1}{4}Q^2)\Phi^{(1)} = \frac{10+d}{4}\kappa\tilde{t}, \quad (5.5)$$

where we have used the same gauge condition with (4.6). In obtaining these equations, we have kept the term up to $O(\kappa)$ and neglected the higher order terms like $O(\kappa^2)$. The above equations, (5.3) \sim (5.5), are different from the case of $\kappa = 0$ in the following two points; (i) The "mass" part of tachyon is shifted from 2 to $2 - \frac{\kappa}{4}$ in (5.3), and this comes from the expansion of T^3 in $v(T)$. Then T^3 term is essential to obtaining the "mass shift" of the tachyon. (ii) Secondly, the couplings among the perturbations of the order κ appear.

Due to the second fact, we must solve these equations simultaneously in order to obtain the dressed factor of the perturbation \tilde{t} . We can solve these in terms of the following ansatz,

$$h_{\mu\nu} = \mu_h\delta_{\mu\nu}e^{\alpha\phi}, \quad \Phi^{(1)} = \mu_\Phi e^{\alpha\phi}, \quad \tilde{t} = \mu_t e^{\alpha\phi}. \quad (5.6)$$

Then we obtain the following relations,

$$\mu_h = \frac{\kappa}{2 - \kappa/4}\mu_t, \quad \mu_\Phi = \frac{10+d}{4}\frac{\kappa}{2 - \kappa/4}\mu_t. \quad (5.7)$$

Then we can see that the right hand side of (5.3) is the order of κ^2 , so we can neglect it in solving (5.3) and we obtain

$$\begin{aligned} \alpha &= \frac{1}{2}(Q - \sqrt{Q^2 - 8 + \kappa}) \\ &= \frac{1}{2}(\sqrt{\frac{25-c}{3}} - \sqrt{\frac{1-c}{3}} + \kappa). \end{aligned} \quad (5.8)$$

This is the main result of our calculation. For $\kappa = 0$, α becomes complex for $c > 1$, but it remains real for $\kappa > 0$ even if $c > 1$. However the reality of α is restricted to the small region of c , $1 + 3\kappa > c$, since κ is small. Then it would be necessary to consider the strong coupling region by some nonperturbative approach in order to see the possibility of extending the region of c where α remains real.

The stability of the system can also be seen by seeing the quadratic terms of the fluctuations $h_{\mu\nu}$, $\Phi^{(1)}$ and \tilde{t} in (4.7). We can assure that there is no tachyonic mode in it for $c < 1 + 3\kappa$.

It can be seen that the above conclusion is sensitive to the term T^3 of $v(T)$. So we should comment on the criticism given before on this term. One is the possibility of rewriting this term by the derivative term like $(\partial^2 T)^3$ in terms of

the field redefinition [6, 7] which is related to the ambiguity of the β -functions due to its renormalization scheme dependence. Generally speaking, the physical consequences should not be influenced by the renormalization scheme. But our result (5.8) could not be obtained if we rewrote the potential. However, the above potential is not obtained by a systematic perturbative expansion, α' expansion, and T^3 term has been added as a non-perturbative term. Then we should be careful in rewriting this potential in terms of the field redefinition due to the renormalization scheme dependence. Then we had kept the form of (2.4) here.

Another criticism [6] is the claim that (2.4) allows the solution, $T = \text{const.}$, and this solution of $T \neq 0$ is in contradiction to the conformal invariance of (2.1). However, the potential $v(T)$ is valid for small T , so we could not say from (2.4) the existence of the solution of $T \neq 0$ which is consistent with the validity of this potential. But this potential leads to a nontrivial solution for T if we add the string-loop corrections to this. In this case, T could take a small vacuum value which is consistent with the assumption of small T expansion. And the problem of the conformal invariance of the 2d action should be reconsidered by taking into account not only of the short distance fluctuations of the field on the world sheet but also of the string loop corrections, so the paradox would be resolved.

6. Concluding remarks

Here we have examined a special quantum surface fluctuation of two-dimensional space-time in terms of the loop-correction of a open string in order to see the vacuum structure of 2d gravity and noncritical string theory. The technique to calculate this correction had been developed in the critical super-string theory by constructing the boundary state accompanying a tube of the closed string. It is applied here to the non-critical string theory and 2d gravity, and this correction leads to the conformal anomaly for the massless channel of the closed string states. Then the field equation of the tachyon is modified for $c = 1$ case where tachyon is massless. As a result, this loop-correction leads to the condensation of the tachyon. Furthermore, the dressed factor of the perturbations around this shifted vacuum becomes real even if c exceeds one. However this range of c where the dressed factor remains real is restricted to a very small region above 1, $c < 1 + 3\kappa$. Since κ denotes the string coupling constant, it would be necessary to consider some non-perturbative approach to see whether we can extend the theory to the region where c is enough large compared to one and to obtain the noncritical string theory of $c > 1$.

References

- [1] F. David, Mod. Phys. Lett. A3(1988)1651; J. Distler and H. Kawai, Nucl. Phys. B321(1989) 509; V. Knizhnik, A. Polyakov and A. Zamolodchikov, Mod. Phys. Lett. A3 (1988)819.
- [2] I.R. Klebanov, I.I.Kogan, and A. Polyakov, Phys. Rev. Lett. 71(1993) 3243; C.Schmidhuber, Nucl. Phys. B404(1993)342; J.Ambjorn and K.Ghoroku, Int. J. Mod. Phys. A9(1994)5689; K.Ghoroku, Phys. Lett. B357(1995)12.
- [3] E.S.Fradkin and A.A.Tseytlin, Nucl. Phys. B261(1985)1; C.G.Callan, D.Friedan, E.J.Martinec and M.J.Perry, Nucl. Phys. B262(1985)593.
- [4] C.Itoi and Y.Watabiki, Phys. Lett. B198(1987)486.
- [5] A.Cooper, L.Susskind and L.Thorlacius, Nucl. Phys. B363(1991)132.
- [6] A.A.Tseytlin, Phys. Lett. B264(1991)311.
- [7] T.Banks, Nucl. Phys. B361(1991)166.
- [8] W.Fishler and L.Susskind, Phys. Lett. B173(1986)262.
- [9] E.S.Fradkin and A.A.Tseytlin, Phys. Lett. B163(1985)123; A.Abouelsaoud, C.G.Callan, C.R.Nappi and S.A.Yost, Nucl. Phys. B280(1987)599; C.G.Callan, C.Lovelace, C.R.Nappi and S.A.Yost, Nucl. Phys. B288(1987)525.
- [10] C.G.Callan, C.Lovelace, C.R.Nappi and S.A.Yost, Nucl. Phys. B293(1987)83; *ibid*, B308(1988)221.
- [11] C.Schmidhuber, "D branes actions", PUPT-1995, hep-th/9601003.
- [12] B.Durhuus, P.Olesen and J.L.Petersen, Nucl. Phys. B198(1982)157;*ibid*, B201(1982)176; B.Durhuus, H.B.Nielsen, P.Olesen and J.L.Petersen, Nucl. Phys. B196(1982)498.
- [13] P.Mansfield, Nucl. Phys. B306(1988)630; P.Mansfield and R.Neves, hep-th/9605097 May 1996.
- [14] V.Dotsenko and V.Fateev, Nucl. Phys. B240(1984)312.
- [15] S.Coleman and E.Weinberg, Phys. Rev. **D7**(1973)1888.